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Laser excitations of atoms can easily cause heating and addition of entropy, however the powerful tool of laser cooling is based on the fact that under specific circumstances they can induce the opposite effect. For example, if an atom absorbs a photon whose momentum is in opposition to its own, its total momentum will decrease. At some later time this photon will be re-emitted from the atom in a random direction, giving the atom a kick. If this atom undergoes many such scattering events there will be a net force on the atom in the direction of the laser, because the random emissions of the photons will average out to zero net change in momentum. This type of cooling is the basis for techniques such as Zeeman slowing and Magneto-optical traps.

On a more basic level, an excited atom is being cooled by emitting a photon into the vacuum. The photon carries away energy and entropy, while the atom descends to a lower energy level. The ability of atoms to readily couple to the photon modes in the vacuum is what makes laser cooling possible.

Recently ideas to simulate this type of coupling in a BEC have been proposed. The atoms to be cooled are placed in contact with a BEC, and the interactions between the two systems causes the BEC to act much like the vacuum in laser cooling. If, as in the case of closed shell atoms, the internal spin of the atom is not affected by the interactions, then this cooling can be said to be coherent.

This paper is a summary of the dynamics of the sympathetic cooling system described above, beginning with a look at the Hamiltonian. A number of simplifying assumptions are then made in order to calculate energy loss rates, and determine the effectiveness of the cooling system. A few corrections to the initial assumptions are noted, and finally an example of Raman-like cooling is provided in the context of this system.

Sympathetic Cooling of Lattice atoms by a Bose-Einstein Condensate

Daniel Schwartz

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August 13, 2010

Laser cooling has become a popular method for enabling studies of quantum interactions. It has brought about various recent experimental achievements, ranging from the creation of Bose-Einstien condensates (BEC)[1], to the quantum teleportation of information over long distances[2]. Despite the variety of experimental techniques currently available to the laser cooling community, there is an ever present need for atoms to be colder. Quantum computation and simulation are two of the driving forces behind this, and are often limited by heating of atoms during gate implementation, or by the need for extremely low temperatures for many body states.

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1 Hamiltonian

Our system has two distinct parts: the superfluid and the lattice atoms. Each of these parts has distinct energetic properties which do not couple to the other, and an interaction term which does. Any other potentials that effect the system are part of a fourth term, denoting external influences. These external potentials will not be addressed directly, but are briefly mentioned at the end. As such the Hamiltonian can be expressed as[3]

$$H_{tot} = H_b + H_a + H_{int} + H_{ext} \tag{1}$$

where H_b represents the energy of the superfluid, H_a is the atoms in the lattice, H_{int} is the interaction between them, and H_{ext} denotes the interaction with external potentials. To understand the interaction dynamics that lead to sympathetic cooling, it is necessary to understand the individual components of the system.

1.1 Lattice atoms

First, consider atoms in a lattice. The light used to create the lattice is typically far detuned from resonance, so these atoms are assumed to be in the ground state. This sub-system has two terms: the potential applied to the atom by the lattice beams, and on-site interactions between lattice atoms.

The potential on the atoms from the lattice beams is[4]

$$H_{la} = \int_{\Re^3} d^3x \hat{\Psi}_a^\dagger(\mathbf{x})(-\frac{\nabla^2}{2m_a} + V_a(x))\hat{\Psi}_a(x) \tag{2}$$

where $\hat{\Psi}_a(x)$ is the field operator, which creates an atom at location x. Subscripts a and b will be used to denote lattice atoms and superfluid atoms respectively. The potential $V_a(x)$ is the lattice standing wave, and can be written[4].

$$\sum_{x_i=x,y,z} V_{a,x_i} Sin^2(\frac{\pi x_i}{d}) \tag{3}$$

where $d = \frac{\lambda}{2}$ is the lattice spacing. Field operators are particularly useful in the context of this discussion because they encode quantum fluctuations into the description of the system. It is often convenient to decompose them into a particular set of modes. Defining $\hat{A}_q^\alpha$ as the annihilation operator for a lattice atom in Bloch band α with quasi-momentum $\hbar q$, and defining $w_q^\alpha(x_i)$ to be the Wannier functions for direction x_i, the field operator can be written

$$\hat{\Psi}_a(x) = \sum_{\alpha_x\alpha_y\alpha_z} \sum_q w_q^\alpha(x) w_q^\alpha(y) w_q^\alpha(z) \hat{A}_q^\alpha \tag{4}$$

The other term is the on-site interactions, which primarily takes the form of density-density interactions. It can be expressed

$$H_{os} = \frac{4\pi a_{aa}}{2m_a} \int_{\Re^3} d^3x \hat{\Psi}_a^\dagger(x)\hat{\Psi}_a^\dagger(x)\hat{\Psi}_a(x)\hat{\Psi}_a(x) \tag{5}$$

where the scattering length between two lattice atoms, a_{aa}, is effectively the strength of the interaction. Note that the field operators mediate this interaction such that if two atoms do not have a spatial overlap in their wave function, their contribution to this term will be zero. This term will not be used much in this analysis because interactions can typically be tuned to be approximately zero to help simplify the system.

1.2 BEC atoms

To model the superfluid, a mean field theory is employed. In a mean field theory, multi-particle systems are treated by calculating average effects of all particles on a single particle. The single particle's effect on the system must then also be averaged in a self consistent way. While this technique causes the loss of spatial correlation data, a BEC does not have any correlations about which to be concerned. It is not derived here, but the Hamiltonian can be written[3]

$$H_{sf} = E_0 + \sum_{q\neq 0} \epsilon(q)\hat{b}_q^\dagger \hat{b}_q \tag{6}$$

The first term, E_0, is the mean superfluid energy. The second accounts for excitations that can exist in the superfluid. Known in this case as Bogoliubov quasi-particles, $\hat{b}$ is their annihilation operator, and $\epsilon(q)$ is their dispersion relation, given in [3] as

$$\epsilon(q) = [\frac{u^2\hbar^2q^2 + (\hbar q)^4}{(2m_b)^2}]^{1/2} = \hbar q[u^2 + \frac{\hbar^2q^2}{(2m_b)^2}]^{1/2} \tag{7}$$

It is a function of $\hbar\mathbf{q}$, the quasi-momentum, and $u = \frac{\sqrt{4\pi\hbar a_{bb}\rho_0}}{m_b}$, the speed of sound in the superfluid. The term a_{bb} denotes the scattering length between superfluid atoms, and ρ_0 is the average density of the superfluid. The latter two parameters are to some extent experimentally adjustable, allowing for limited control of the excitation modes in the superfluid. As discussed later, the spectrum will be linear for low q, and quadratic for high q, causing two distinct behavior regimes.

1.3 Interactions

Interactions between the superfluid and the lattice atoms take the form of density-density interactions, which can be viewed in terms of emission into a reservoir. Consider a lattice atom in a non-zero energy state. If this atom collides with a superfluid atom, it can transfer energy and momentum, and create an excitation in the superfluid. It will then decay to a lower state. In the case of the vacuum, atoms would emit a photon, while in this system atoms emit something more like a phonon. Our model's Bogoliubov excitation modes are the phonon-like emissions that will mediate the transfer of energy. This analogy to spontaneous emission can be extended further. If an excitation exists in the superfluid it can be absorbed by a lattice atom, or cause stimulated emission of a second excitation.

A quantitative explanation requires the use of the density fluctuation operator. This operator, denoted by $\delta\hat{\rho}(x)$ is used with the mean field approximation to express a local increase or decrease in the density of atoms at a point. The full density operator would then be

$$\hat{\rho}(x) = \rho_0 + \delta\hat{\rho}(x) \tag{8}$$

and returns the density at any point in the relevant system.

Since there are two systems, there are also two separate density operators. The superfluid has some mean density, and a density fluctuation takes the form of an excitation. The lattice atoms are modeled as their spatial wave functions, localized by the lattice potential.

When an excitation and a lattice atom interact their spatial wave functions must overlap, allowing a collision to occur. The interaction term must therefor be proportional to the overlap integral between the two density fluctuation operators. Additionally, the strength of the interaction depends on the scattering length between the two different types of atoms, a_{ab}. The final interaction potential is then [3]

$$H_{int} = \frac{4\pi\hbar^2 a_{ab}}{2\mu} \int \delta\hat{\rho}(r)\delta\hat{\rho}_{atom}(r)d^3r \tag{9}$$

where μ is the reduced mass between lattice and superfluid atoms. This is the final term in the Hamiltonian of this system, and encompasses most of the interesting physics in this discussion. This term will be shown to cause transitions, meaning that it has non-zero off diagonal matrix elements

2 Dissipation

There are two distinct types of dissipation that need to be considered. The first is atoms moving between Bloch bands, and will be the focus of most of the analysis. It will be explained in the context of flat Bloch bands for simplicity. The second type is atoms moving within the Bloch band. This type of dissipation is mostly suppressed under typical experimental conditions due to conservation laws, but will be touched upon briefly.

The analysis will first make several assumption about the system, then proceed to write down the simplified Hamiltonian. Next, Fermi's golden rule calculations are performed, and finally corrections are made to the analysis.

2.1 Assumptions

The initial analysis of the Hamiltonian for this system is very general, and it is useful to make several assumptions before proceeding to solve for the behavior of the system.

As mentioned before, on site interactions will be taken to be zero. By making the scattering length $a_{aa} = 0$, the system is greatly simplified, and reduces to a single integral.

Next, assume that $E << V_a$. This is known as the deep lattice potential assumption, and qualitatively means atoms will be near the bottom of the lattice site. In addition, the non-tunneling assumption is made, which amounts to saying that $E_R << V_a$ where E_R is the recoil energy. Because the trap potential goes as Sin^2kx, the localization allows a Taylor expansion about $x = 0$ to be truncated to lowest order. The atoms thus feel only a harmonic potential under these conditions.[3]

$$H_a = \int_{\Re^3} d^3x \hat{\Psi}_a^\dagger(\mathbf{x})(-\frac{\nabla^2}{2m_a} + \sum_{i=x,y,z} V_{a,i}(\frac{\pi i}{d})^2)\hat{\Psi}_a(\mathbf{x}) \tag{10}$$

The harmonic oscillator assumption simplifies the system because eigenstates for lattice atoms are just the Fock states, denoted as $|n\rangle$ or $|m\rangle$. Note that since the three dimensions are not coupled in the harmonic oscillator solution, it is valid to consider a one-dimensional system which can later be generalized.

Inter-Bloch band transitions in our lattice system must therefore be between harmonic oscillator levels. The goal of the Fermi's golden rule calculations mentioned before will be to calculate the relaxation rate of the harmonic oscillator states in the presence of the bath.

To simplify the superfluid portion of the system, the zero temperature limit can be taken. Zero temperature in this case applies to the excitation modes, and means that $b_q^\dagger b_q$ is zero for all $q \neq 0$. Unlike the previous assumptions, zero temperature is not a reasonable experimental approximation for the equilibrium vibration population. It will therefore will need to be addressed later, however it is useful for calculating relaxation rates.

Finally, the interspecies interaction potential is assumed to be small such that perturbation theory can be applied. This can be accomplished by tuning the scattering length a_{ab} with a Feshbach resonance, or by careful choice of atomic species. For simplicity, the masses of the lattice and superfluid atoms are assumed to be equal.

The interaction term still needs to be simplified to an integrable equation. As mentioned in the discussion of the Hamiltonian, lattice atom density fluctuation operators are just the square of the spatial wave functions of the individual atoms. In the limit where the atoms sit in a deep well without tunneling, the wave function will be well localized in space, and can be approximated as a delta function. Equation 9 now simplifies to[3]

$$H_{int} = \frac{4\pi\hbar^2 a_{ab}}{2\mu}\delta\hat{\rho}(r_0) \tag{11}$$

where r_0 is the location of the lattice atom, and $\delta\hat{\rho}$ denotes the superfluid density fluctuation operator. This is a restatement of the fact that collisions cannot occur except at locations where lattice atoms exist.

For the normalization used here, the operator $\hat{\Psi}^\dagger(x)\hat{\Psi}(x)$ returns the number of particles at x, while $\hat{\rho}(x)$ returns the density at x. This means that $\hat{\rho}(x) = \frac{1}{V} * \hat{\Psi}^\dagger(x)\hat{\Psi}(x)$[3] where V is the relevant volume.

Rewriting the field operator using equation 8

$$\hat{\Psi}(x) = \sqrt{\rho_0} + \delta\hat{\Psi}(x) \tag{12}$$

which is expressed in terms of the mean density. The new term introduced above, $\delta\hat{\Psi}(x)$, is the fluctuation in the field operator. As mentioned, the field operator contains information on both the classical and quantum elements of the system. A superfluid is essentially a classical object, which is why it can be effectively described by a mean field theory. The quantum fluctuations in a superfluid can therefore be treated as a small perturbation added to the classical portion of the operator. This term must directly equate to the density fluctuation operator, as they are both non-classical. The number operator can be expended using this notation to be

$$\hat{\Psi}^\dagger(x)\hat{\Psi}(x) = \rho_0 + \sqrt{\rho_0}(\delta\hat{\Psi}(x) + \delta\hat{\Psi}^\dagger(x)) + \delta\hat{\Psi}(x)\delta\hat{\Psi}^\dagger(x) \tag{13}$$

where the latter terms must be equal to $\delta\hat{\rho}(x)$.

As mentioned in the Hamiltonian, fluctuations in the superfluid field take the form of Bogoluibov excitations. The fluctuation operator can then be expressed as[3]

$$\delta\hat{\Psi}(x) = \frac{1}{\sqrt{V}}\sum_q (u_q\hat{b}_q \exp \imath\mathbf{q}\cdot\mathbf{r} + v_q\hat{b}_q^\dagger exp{-}\imath\mathbf{q}\cdot\mathbf{r}) \tag{14}$$

where

$$u_q = v_q * L_q = \frac{L_q}{\sqrt{1 - L_q^2}} \tag{15}$$

and

$$L_q = \frac{\epsilon_q - \frac{(\hbar q)^2}{2m_b} - m_b u^2}{m_b u^2} \tag{16}$$

Translating to words, each interaction between the lattice and superfluid must, to first order, be mediated by a single excitation or de-excitation of the superfluid interacting with a single lattice atom. The terms u_q and v_q are the standard Bogoliubov rotation factors, while L_q comes from conservation laws.

2.2 Fermi's Golden Rule

The system has now been reduced to a perturbed Hamiltonian. The unperturbed portion is equivalent to an atom in a harmonic oscillator, and thus working in the Fock basis is preferable. Much as an excited atom coupling to a vacuum, this perturbation causes the lattice atoms to jump between Fock states. By this analogy the transition rates are needed to fully understand the cooling process.

Consider an arbitrary level $|m\rangle$. The change in occupation probability $\dot{p}_m$ must be proportional the frequency of the atoms decaying from a higher level to m, added to that of the atoms decaying from m to a lower level. The frequencies are denoted as $F_{n\to m}$ and $F_{m\to n'}$ respectively. Summing over all possible transitions we obtain[3]

$$\dot{p}_m = \sum_{n'>m} F_{n'\to m} p'_n - \sum_{m>n} F_{m\to n} p_m \tag{17}$$

Calculating F coefficients is done by noting that the dispersion relation in the lattice and in the superfluid are not the same. The only time an excitation can be created is when energy conservation is satisfied, placing restrictions on F. Both F values must therefore include $\delta(\hbar\omega(n-m)-\epsilon(q))$ as a multiplicand, where $\epsilon(q)$ is the superfluid dispersion relation from equation 7.

The transition strength will be proportional to
$|\langle m|H_{int}|n\rangle|^2$, and can be calculated from perturbation theory. This matrix element is exactly solvable because Fock states can be expressed in terms of Hermite polynomials. In the zero temperature limit it should also be noted that since no excitations exist, the term in the interaction proportional to b_q must go to zero. The transition rate reduces to solving the equation[3]

$$F_{n\to m} = \frac{2\pi}{\hbar}\sum_{\mathbf{q}} |\langle n|\frac{1}{\sqrt{V}} u_q \hat{b}_q^\dagger \exp \imath\mathbf{q}\cdot\mathbf{r}|m\rangle|^2 \tag{18}$$

for values of q where

$$\epsilon(q) = \hbar\omega(n-m) \tag{19}$$

The rate is solved by utilizing a useful identity given in [3]

$$\langle m|e^{\imath p_x x}|n\rangle = \sqrt{\frac{m!}{n!}} e^{-l_0^2 q_x^2/4} (\frac{-\imath l_0 q_x}{\sqrt{2}})^{n-m} L_m^{n-m}(\frac{l_0^2 q_x^2}{2}) \tag{20}$$

where l_0 is the standard harmonic oscillator length

The average over initial states is taken in that we assume the system to start in state m, and the sum over final states is the sum over the quasi-momenta of excitations.

To gain a qualitative understanding of this system, it is useful to consider the high and low energy limits for excitations. The relevant reference energy is that of a superfluid particle moving at the speed of sound in the superfluid. The high and low energy regimes are thus appropriately named supersonic and subsonic respectively.

2.3 Supersonic Regime

This regime can be represented quantitatively by saying $\hbar\omega_{nm} >> m_b u^2/2$ where u is the speed of sound in the superfluid, and ω_{nm} is the harmonic oscillator spacing. The speed of sound is a function of the superfluid density and scattering length, however typical experimental parameters make this regime the most likely to be observed. [3]

Using equation 7 with the high energy approximation the dispersion relation simplifies to $\epsilon(q) \approx \frac{\hbar^2 q^2}{2m_b} + U_0$, similar to a free particle except for U_0. This is the energy shift caused by the mean field of the superfluid. Qualitatively, because the excitations move so quickly through the superfluid, the other atoms do not react to them. Unless it collides with another lattice atom, this excitation will exit the superfluid, carrying away entropy and energy.

Quantifying the effectiveness of the cooling is done by calculating the rate of energy change[3]

$$\dot{\epsilon}(n) = \sum_{n=0}^{n\le m} (E_m - E_n) F_{m\to n} \tag{21}$$

where E_i is the energy of the lattice atom in state i. The values for $F_{m\to n}$ have been computed numerically by [3]. The result is that the transition rate between two levels is not a strong function of the final level. The energy decay

rate can be written as[3]

$$\dot{\epsilon}(n) = -\frac{g_{ab}^2 \rho_0 m^{3/2}}{\pi\hbar^2\sqrt{2}}\alpha[\epsilon(n)]^{3/2} \tag{22}$$

where $\alpha = .3$ and $g_{ab} = \frac{4\pi\hbar^2 a_{ab}}{2\mu}$. The important result is that the decay of energy is not exponential in time, and will remain significant for transitions from the first excited state to the ground state.[3] Sympathetic cooling in the supersonic regime is therefore an effective way to reach the ground state of a lattice site.

It is helpful to define a single value which quantifies the cooling rate. For this purpose [3] uses $F_{1\to 0}$, the rate of decay from the first excited state to the ground state. Equation 18 shows that for lower energy levels the rate of energy loss will be less. This transition is therefore the slowest, and can be used to characterize the system. The result is[3]

$$F_{1\to 0} = 0.3789\frac{g_{ab}^2 \rho_0 m}{\pi\hbar^3 l_0\sqrt{2}} \tag{23}$$

where l_0 is the characteristic harmonic oscillator length.

2.4 Subsonic Regime

At the other end of the energy spectra, the approximation $\hbar\omega_{nm} << m_b u^2/2$ can be made. Following the same logic as the supersonic regime, the dispersion relation becomes $\epsilon(q) \approx \hbar q u$. Low energy excitations thus take the form of phonons.

The transition rates can again be calculated and used to solve for the rate of energy decay. This time,[3]

$$\dot{\epsilon}(n) = -\frac{g_{ab}^2 \rho_0 \omega^4}{12\pi m_a m_b u^7} * \epsilon(n) \tag{24}$$

which when solved will have an exponential dependance on the energy. In addition, solutions to equation 24 given the subsonic approximation lead to transition rates that are significant only when $n = m - 1$, meaning atoms in the lattice will decay predominantly into the energy level directly below them. The exponential suppression of the energy decay coupled with the inability to decay to the ground state from higher levels make this regime undesirable for sympathetic cooling.

3 Corrections

3.1 Zero-Temperature

Previously the superfluid was assumed to have an effective temperature of zero. In practice, excitations in the superfluid will limit the cooling rate, and set a minimum possible temperature for the sympathetic cooling. To add this effect to the rate equations (17), an additional term is needed. It can be written[3]

$$\sum_n H_{n,m}(p_n - p_m) \tag{25}$$

where

$$H_{n,m} = \frac{2\pi}{\hbar}\sum_{\mathbf{q}} N(\mathbf{q})|\langle n|H_{int}|m\rangle|^2\delta(\hbar\omega|n-m| - \epsilon(q)) \tag{26}$$

Note first that the sum is over all states $|n\rangle$, not just the lower or higher states is taken. This change is because a thermal excitation could either be absorbed, or could stimulate the emission of another excitation in the same mode. The same reasoning applies to the change to the delta function, because transitions to higher energy levels are now allowed by energy conservation. The rate $H_{n,m}$ is naturally also proportional to the number of thermal excitations in each mode, $N(\mathbf{q})$. Finally, the rate is multiplied by the difference in the probabilities of being in state $|n\rangle$ and state $|m\rangle$. This multiplicand can be understood as determining whether stimulated emission or absorption is the more likely process, and must be proportional to the population difference. By considering both emission and absorption processes there is no need to correct the values of $F_{n\to m}$ where it was assumed $b_q|n\rangle = 0$

The approximation made earlier which ignored $H_{n,m}$ amounts to saying that $\epsilon(q) \gg k_b T$. In practice there will be a small correction to the system due to finite temperature, resulting in the lowest achievable energy in the system having a non-zero population in the excited states. These excited states will be populated according to a Boltzmann distribution. The probability of being in the ground state for low temperatures reduces to[3]

$$\overline{p_0} = 1 - exp[\frac{-\hbar\omega}{k_b T}] \tag{27}$$

Typical experimental parameters discussed in [3] suggest these thermal occupations will be small, however they could become larger as the superfluid absorbs more energy. This heating would depend on a number of factors, including the size of the superfluid and lattice systems, the initial energy levels of the lattice atoms, and the regime of the excitations. The superfluid will therefore need to undergo continuous evaporation in order to maintain low temperatures.

3.2 Bloch Structure

In the beginning of this section lattice atoms were approximated to be in a harmonic potential. For a more thorough understanding of the system, the Bloch structure needs to be considered. Previously the energy of the atoms was $\epsilon_n = \hbar\omega(n + \frac{1}{2})$. In a Bloch band this is replaced by [4]

$$\epsilon_q^\alpha = -2J^\alpha \cos qd \tag{28}$$

where $d = \frac{\lambda}{2}$ is the lattice spacing, and J^α is characteristic intra-band energy for band α. Note the dependance on the new quantum number, the quasi-momentum q, not present in the previous analysis.

In the Bloch band picture each harmonic level has now become a continuous band of levels. Atoms can travel within the Bloch band, meaning changes in momentum and energy much smaller than the band separation are allowed. These transitions can lead to excitations in the superfluid on a much smaller scale than the Bloch level separation.[4] The excitations typically fall into the subsonic regime, and thus momentum conservation requires $\delta k = q$, and energy conservation requires[4]

$$-2J^0(\cos kd - cosk'd) \approx uq \tag{29}$$

Here k and k' are the momentum of the lattice atom inside the Bloch band before and after the transition respectively, and $k' = k + q$. For $k \approx 0$ the left side can take on values between $-4J^0$ and zero. Intra-band heating can then be minimized if $u \gg -4J^0$, except for small k values.[4] This will result in the suppression of the finite temperature heating effects within the Bloch bands, which will prove to be important in the next portion of the analysis.

3.3 Cooling Limits

All cooling schemes have some limit to which they can cool atoms. In a laser cooling system, this limit is set by the energy of the photons which atoms emit during the cooling process, and is known as the recoil limit. Limit is not a perfect description of the effect however, because techniques exist to cool below the recoil limit. In this sympathetic system the effective 'recoil limit' is the energy of the excitations produced by the lattice atoms. This limit will be significantly lower than the laser cooling recoil limit, because the energy of excitations is much less than that of laser cooling photons. Nonetheless, techniques have been postulated to produce atoms below the limit in this system as well.

One such method is described in [4]. As described above, in the Bloch band structure a range of values for q are allowed. Under normal circumstances when a lattice atom decays to the ground Bloch state, it will have some probability to land in any quasi-momentum state. Once the atom has a particular quasi-momentum in the ground Bloch state, it will not cool further because transitions along the Bloch band are suppressed by conservation laws.

As suggested in [4], to overcome this limitation atoms need only be excited to a higher Bloch band selectively, and then be allowed to decay normally. Consider the addition of Raman excitation to the Hamiltonian in a one dimensional system. Such a term appears as[4]

$$H_{raman} = \frac{\Omega}{2} \sum_q ((\hat{A}_q^1)^\dagger \hat{A}_{q-\delta q}^0 + H.c.) \tag{30}$$

where Ω is the effective Rabi frequency, and A_q^α is the same momentum annihilation operator described in equation 4. This term takes a particle in the ground Bloch band with momentum $q - \delta q$ and creates a particle in the first excited band with momentum q. The effective Rabi frequency can be written as[4]

$$\Omega = \Omega_R \int exp(-\imath \delta q x) w^1(x) w^0(x) dx \tag{31}$$

where Ω_R is the time dependant two photon Rabi frequency, and $w^\alpha(x)$ represents the spatial distribution of the lattice atoms (Wannier function) in one dimension. In the deep lattice limit, these become the familiar harmonic oscillator levels from earlier. The momentum change δq will be a function of the detuning of the Raman beams, which can be written as δ.

The probability that an atom with a given quasi-momentum will make a transition to the first Bloch band is proportional to the Raman detuning and the length of time that the light is present. By careful choice of these variables it is possible to minimize the probability of $q = 0$ atoms making the transition, while simultaneously allowing some set of non-zero quasi-momentum atoms to become excited. By creating a series of these pulses, all non-zero quasi-momentum atoms can be excited, save some small group around zero.

The excited atoms will then decay back to the ground state. The change in q caused by the spontaneous emission of a phonon into the three dimensional superfluid will cause the decaying atom to randomly populate some new quasi-momentum state. Because the process is random, there is a chance that these atoms will land near zero quasi-momentum, where excitations are very unlikely to occur.

As this process is repeated, the number of atoms near $q = 0$ will increase until it is the only significant population. The atoms will then have a finite width in momentum space, which can be used to define a temperature for the atoms as[4]

$$k_B T = 2J_0(\Delta q d)^2 \tag{32}$$

with Δq being the full width half max of the momentum distribution.

The limitations on this cooling scheme are twofold. First, atoms near $q = 0$ have some small but non-zero probability of undergoing a Raman excitation. This will create a small recycling rate, where cold atoms are heated. In contrast, the rate of cooling is proportional to the number of atoms outside the dark region, causing it to decrease as the atoms get colder.

Secondly, atoms may undergo absorption of phonon excitations from the superfluid. Due to conservation laws, these excitations will not cause motion along the bands, but rather to an excited Bloch state. As such, it can be viewed as sympathetic heating and will again create a small recycling rate. These two rates will balance against the cooling rate to set the minimum achievable temperature. Actual reheating rates for this system have not yet been calculated as they involve higher order scattering calculations, which can be a significant challenge. Minimum temperatures should be substantially lower than those achievable by sympathetic cooling.

In conclusion, using a BEC as a superfluid reservoir provides significantly lower temperature than laser cooling alone. Interest in this topic has also spawned several theoretical papers [5][6][7] with possible uses for these systems in quantum computing.

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